

This is Gruber's alternate solution to his one that was printed in *The Washington Post* and in other papers.

CLARIFICATION

Expanded answer to genius quiz

Some brain teasers can keep even whiz kids scratching their heads for weeks.

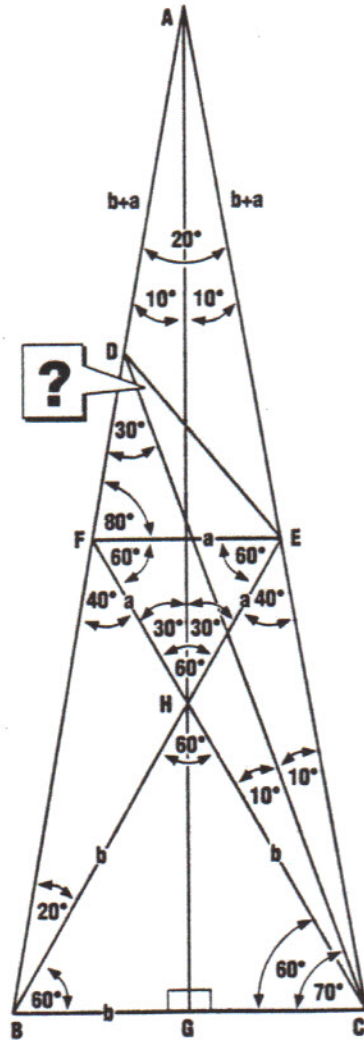
That's what we discovered when we ran Dr. Gary Gruber's Genius Quiz on Sept. 5. Newspapers all over the country that ran the quiz received complaints about question 12. Even college math teachers complained that the question did not give enough information to arrive at the answer. Many of those complaints were forwarded to Gruber's office in California, and Gruber says he is still getting mail almost two months after the original quiz ran.

So if you were one of the many people stumped by question 12, here is a repeat of the original question, the original answer, and an expanded explanation by Gruber.

QUESTION: Suppose you are given a triangle ABC with sides AB equals AC. Draw a line from C to side AB. Call the line CD. Now draw a line from B to side AC. Call that line BE. If angle EBC equals 60 degrees, angle BCD equals 70 degrees, angle ABE equals 20 degrees and angle DCE equals 10 degrees, find what angle EDC is. (Do not do this trigonometrically; do it geometrically to get an exact answer.)

(A) 10 degrees (B) 15 degrees (C) 20 degrees (D) 25 degrees (E) none of these.

ORIGINAL ANSWER: (C) Use the fact that angle ABE is equal to 20 degrees and angle A is equal to 20 degrees to make the triangle AEB isosceles.



EXPLANATION:

Begin with triangle ABC. Label angles using the fact that the sum of the interior angles of a triangle is 180 degrees.

1. Draw EF parallel to BC. The angle DFE = 80 degrees because of equal corresponding angles of parallel lines.

2. Drop a perpendicular line to BC from A hitting BC at G. Because of congruent triangles ABG and AGC, angle BAG = angle CAG = 10 degrees.
3. Line AG passes through point H which is the vertex of triangle FEH, because of our symmetry.
4. Angle BHC = 60 degrees since the rest of the angles of the triangle BHC are both 60 degrees.
5. BE = FC (because of corresponding sides of congruent triangles FBC and ECB. BH = HC (call BH = b) because triangle BHC is isosceles. So by subtraction, FH = HE.
6. Since angle FHE = 60 degrees (vertical angle to BHC), and because FH = HE from #5, angle FHE = angle HEF = 60 degrees, so triangle FHE is equilateral. Thus FE = FH = HE. Call each of those sides "a."
7. Now AF = AE (Because AB = AC and FB = EC, by subtraction AF = AE).
8. Because triangle AEB is isosceles, AE = BE = b + a. Thus AF = BE = b + a (since AF = AE from #7).
9. BE = FC (congruent triangles BEC and BFC), so AF = FC, since AF = BE from #8.
10. Now watch this: Triangle AFH is congruent to triangle CFD because AF=FC, angle AFH = 140 degrees = angle CFD, angle DCF = 10 degrees = angle FAH. Thus corresponding sides of the congruent triangles AFH and triangle CFD are equal, so FH = FD. But FH = FE from #6, so FE = FD.
11. Since FE = FD, angle FDE = angle FED and since angle DFE = 80 degrees from #1, angle FDE = 50 degrees = angle FED.
12. But angle FDC = 30 degrees, so by subtraction, angle EDC = 20 degrees!